Semester Two Examination, 2023

Question/Answer booklet

MATHEMATICS SPECIALIST UNITS 3&4

Section Two: Calculator-assumed

SOLUTIONS

WA student number: In figures

In words

Your name

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	48	35
Section Two: Calculator-assumed	12	12	100	90	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (90 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8 **(6 marks)**

A small helium balloon is released and rises vertically so that its height h metres above its launch site after t seconds is given by $h = 1.2t^{1.5}$. A video camera is located 36 metres horizontally from the launch site of the balloon and automatically rotates so that it is always pointing directly at the balloon.

Determine the rate at which the camera is rotating 9 seconds after the balloon is released.

Solution

Since $h = 1.2t^{1.5}$ then given rate is $\frac{dh}{dt} = 1.8\sqrt{t}$.

Required rate is $\frac{d\theta}{dt}$ and the relation between variables is $h = 36\tan\theta$.

$$\frac{dh}{d\theta} = 36\sec^2\theta$$

$$\frac{d\theta}{dt} = \frac{d\theta}{dh} \times \frac{dh}{dt}$$
$$= \frac{1.8\sqrt{t}}{36\sec^2\theta}$$

When $t = 9, h = 1.2(9)^{1.5} = 32.4$ and so $\tan\theta = 32.4 \div 36 = 0.9$ and then $\sec^2\theta = 1 + 0.9^2 = 1.81$. (NB $\theta \approx 42°$ or 0.732^{r}). Hence:

$$\frac{d\theta}{dt} = \frac{1.8\sqrt{9}}{36 \times 1.81} = \frac{15}{181} \approx 0.083$$

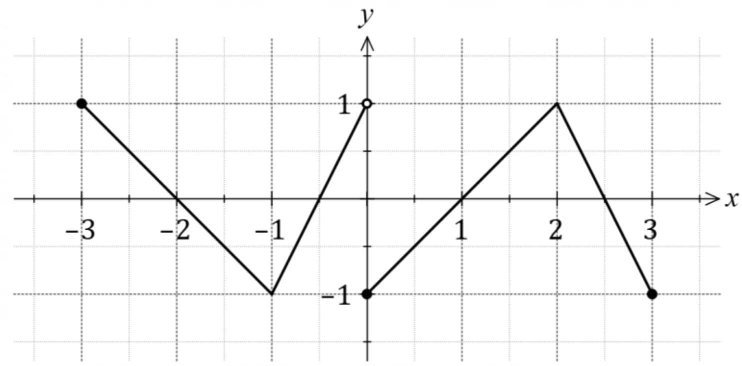
The camera is rotating at 0.083 radians per second.

Specific behaviours

- ✓ differentiates to obtain given rate
- ✓ obtains formula for h in terms of θ
- ✓ obtains formula for required rate in terms of t and θ
- ✓ obtains value for θ (or $\tan\theta$) at required time
- ✓ substitutes values into required rate formula
- ✓ states correct rate, with units.

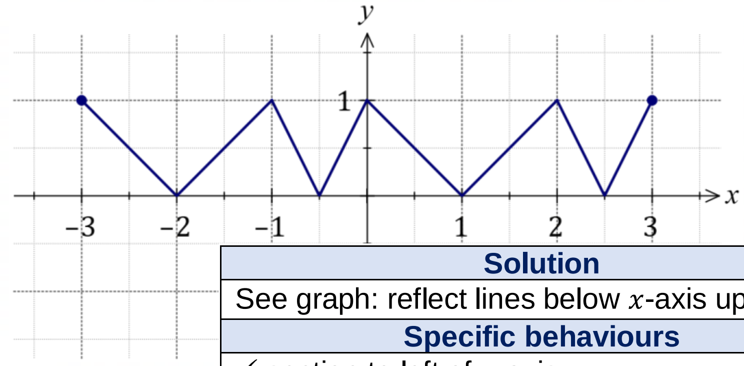
Question 9 **(6 marks)**

The graph of $y = f(x)$ is shown.



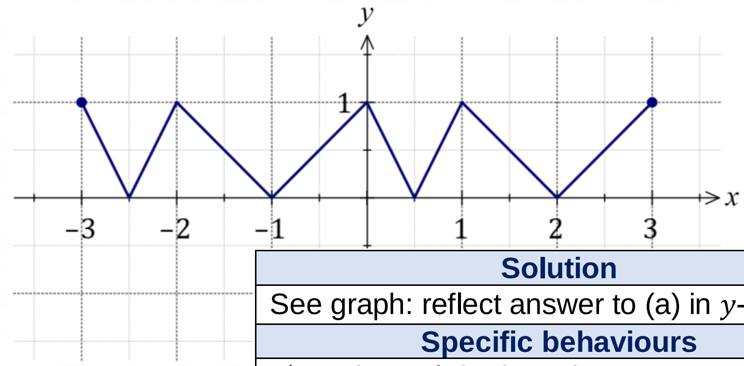
Using the set of axes provided, draw the graph of

(a) $y = |f(x)|$. (2 marks)



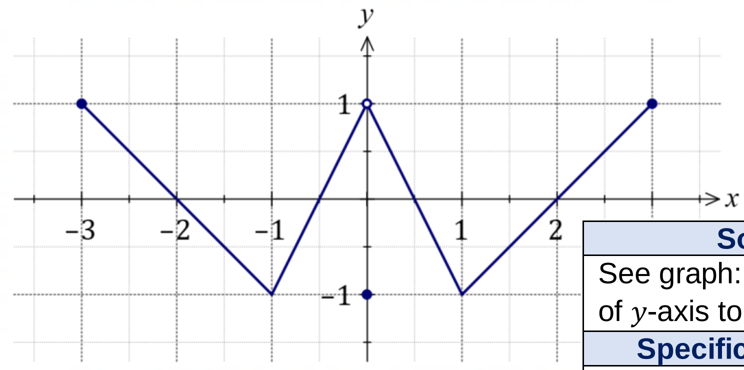
Solution
See graph: reflect lines below x-axis upwards
Specific behaviours
✓ section to left of y-axis
✓ section to right of y-axis

(b) $y = |f(-x)|$. (2 marks)



Solution
See graph: reflect answer to (a) in y-axis
Specific behaviours
✓ section to left of y-axis
✓ section to right of y-axis

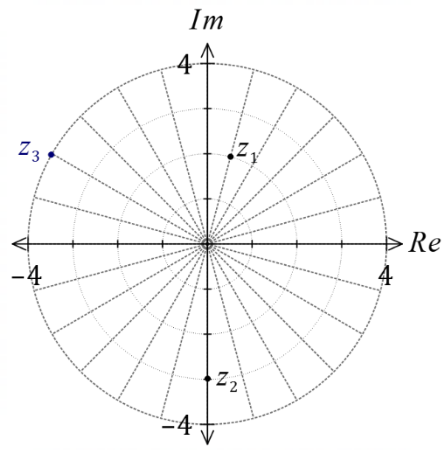
(c) $y = f(-|x|)$. (2 marks)



Solution
See graph: reflect lines left of y-axis to right
Specific behaviours
✓ lines as shown
✓ features when $x = 0$

Question 10 **(9 marks)**

The complex numbers z_1 and z_2 are shown in the complex plane at right.



Solution (c)
See diagram for z_3
Specific behaviours
✓ correct magnitude
✓ correct argument

(a) Express z_1 in polar form. (1 mark)

Solution
$z_1 = 2\,\mathrm{cis}\,\dfrac{5\pi}{12}$
Specific behaviours
✓ correct expression

(b) Express z_2 in Cartesian form. (1 mark)

Solution
$z_2 = -3i$
Specific behaviours
✓ correct expression

(c) Plot z_3 in the complex plane above, given that $z_3 = z_1^2$. (2 marks)

(d) Determine the argument of z_4 when $z_4 = (3 + z_2)(-\sqrt{3} + i)$. (2 marks)

Solution
$\arg(3 - 3i) = -\dfrac{\pi}{4}, \qquad \arg(-\sqrt{3} + i) = \dfrac{5\pi}{6}$ $\arg(z_4) = -\dfrac{\pi}{4} + \dfrac{5\pi}{6} = \dfrac{7\pi}{12}$
Specific behaviours
✓ indicates one correct argument of factors
✓ correct argument

(e) Let $w = a\,\mathrm{cis}\,\phi$. Express $\dfrac{z_1}{w^2(1-i)}$ in polar form in terms of the real constants a and ϕ. (3 marks)

Solution
$\dfrac{z_1}{w^2(1-i)} = 2\,\mathrm{cis}\left(\dfrac{5\pi}{12}\right) \div \left(a^2\,\mathrm{cis}(2\phi) \times \sqrt{2}\,\mathrm{cis}\left(-\dfrac{\pi}{4}\right)\right)$ $= \dfrac{2}{a^2\sqrt{2}}\,\mathrm{cis}\left(\dfrac{5\pi}{12} - 2\phi + \dfrac{\pi}{4}\right)$ $= \dfrac{\sqrt{2}}{a^2}\,\mathrm{cis}\left(\dfrac{2\pi}{3} - 2\phi\right)$
Specific behaviours
✓ expresses all terms in polar form
✓ correct magnitude in terms of a
✓ correct argument in terms of ϕ

Question 11 **(7 marks)**

A factory advertises that its hand-held sparklers burn for an average of 38 seconds. The standard deviation of the burn times is known to be 3.2 seconds.

(a) Quality control took a random sample of 60 sparklers from the factory production line and recorded their burn times. These times were used to calculate the P percent confidence interval for the population mean burn time as $36.82 \le \mu \le 38.18$ seconds. Determine the value of P. (3 marks)

Solution
$se = \frac{3.2}{\sqrt{60}} = 0.4131, \quad E = \frac{38.18 - 36.82}{2} = 0.68$ $z = 0.68 \div 0.4131 = 1.646 \Rightarrow P = 90$
Specific behaviours
✓ indicates standard error or margin of error   ✓ forms equation for z-score   ✓ correct value of P

A consumer watchdog tested a random sample of 48 sparklers made by the factory and their mean burn time was 36.7 seconds.

(b) Describe and construct a suitable interval estimate based on this sample that can be used to advise the watchdog on the reasonableness of the factory's advertising and use the interval estimate to provide that advice. (4 marks)

Solution
A suitable interval estimate is a 95% confidence interval for the population mean burn time of a candle that can be constructed using the sample mean and known population standard deviation: $se = \frac{3.2}{\sqrt{48}} = 0.4619, \quad z_{0.95} = 1.96, \quad E = 0.4619 \times 1.96 = 0.9053$ Interval estimate for population mean: $36.7 \pm 0.9053 \to 35.79 < \mu < 37.61$ Advice to watchdog is that because the interval estimate for the mean burn time of all sparklers does not contain the advertised time of 38 seconds, then the factory's advertising is not reasonable.   *Other intervals:* $90\% \to 35.94 < \mu < 37.46$ *and* $99\% \to 35.51 < \mu < 37.89$.
Specific behaviours
✓ correctly describes interval for population mean using $90\% \le$ confidence level $\le 99\%$ ✓ calculates variance of sampling distribution   ✓ constructs an interval estimate for population mean   ✓ advises watchdog, with reasoning, that advertised average is not reasonable

Question 12 **(7 marks)**

(a) Show that $\dfrac{5}{x-2}+\dfrac{4}{x^2+4}=\dfrac{5x^2+4x+12}{x^3-2x^2+4x-8}$. (1 mark)

Solution
$\text{LHS}=\dfrac{5}{x-2}+\dfrac{4}{x^2+4}$ $=\dfrac{5x^2+20+4x-8}{(x-2)(x^2+4)}$ $=\dfrac{5x^2+4x+12}{x^3-2x^2+4x-8}=\text{RHS}$
Specific behaviours
✓ combines into single fraction with expanded numerator

(b) Hence show that $\displaystyle\int_{-2}^{0}\left(\frac{5x^2+4x+12}{x^3-2x^2+4x-8}\right)dx=\frac{\pi}{2}-5\ln 2$, using the substitution $x=2\tan\theta$ where appropriate. (6 marks)

Solution

$$I=\int_{-2}^{0}\left(\frac{5x^2+4x+12}{x^3-2x^2+4x-8}\right)dx=\int_{-2}^{0}\frac{5}{x-2}dx+\int_{-2}^{0}\frac{4}{x^2+4}dx$$

$$\int_{-2}^{0}\frac{5}{x-2}dx=[5\ln|x-2|]_{-2}^{0}$$
$$=5\ln 2-5\ln 4=-5\ln 2$$

When $x=2\tan\theta$:

$$dx=2\sec^2\theta\,d\theta;\quad x^2+4=4\tan^2\theta+4=4\sec^2\theta;\quad x=0,\theta=0;\quad x=-2,\theta=-\frac{\pi}{4}$$

$$\int_{-2}^{0}\frac{4}{x^2+4}dx=\int_{-\frac{\pi}{4}}^{0}\frac{4\times 2\sec^2\theta}{4\sec^2\theta}d\theta$$
$$=\int_{-\frac{\pi}{4}}^{0}2\,d\theta$$
$$=[2\theta]_{-\frac{\pi}{4}}^{0}=\frac{\pi}{2}$$

Hence $I=\dfrac{\pi}{2}-5\ln 2$.

Specific behaviours

- ✓ antidifferentiates first integral, using absolute value brackets
- ✓ substitutes and simplifies to obtain $-5\ln 2$
- ✓ uses substitution to relate dx and $d\theta$
- ✓ adjusts bounds of integration
- ✓ writes and simplifies second integral in terms of θ
- ✓ antidifferentiates and substitutes to obtain $\dfrac{\pi}{2}$ and hence I

Question 13 **(9 marks)**

The Cartesian equation of sphere S is $(x-5)^2+(y+3)^2+(z-3)^2=9$.

(a) State the vector equation of sphere S. (1 mark)

Solution
$\left\lvert \underset{\sim}{r} - \begin{pmatrix} 5 \\ -3 \\ 3 \end{pmatrix} \right\rvert = 3$
Specific behaviours
✓ correct equation

The position vector of particle P at time t seconds is given by $\underset{\sim}{r}(t) = \begin{pmatrix} -2 \\ -7 \\ 8 \end{pmatrix} + t\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$.

(b) Show that the path of P is tangential to sphere S. (3 marks)

Solution
Path of P in parametric form: $x=-2+2t, \quad y=-7+2t, \quad z=8-t$ Substituting for x, y, z into equation of sphere: $(-2+2t-5)^2+(-7+2t+3)^2+(8-t-3)^2=9$ Expanding and solving this equation: $9t^2-54t+81=0$ $(t-3)^2=0 \Rightarrow t=3$ Since there is exactly one solution to this quadratic equation, then the path of particle must be tangential to sphere.
Specific behaviours
✓ substitutes for x, y, z in sphere equation ✓ simplifies equation ✓ explains meaning of exactly one solution

Particle Q is moving with a constant velocity and has position vector $\begin{pmatrix} 4 \\ 11 \\ -5 \end{pmatrix}$ when $t = 0$. Two seconds later, its position vector is $\begin{pmatrix} 6 \\ 7 \\ -1 \end{pmatrix}$.

(c) Show that the paths of P and Q cross but that they do not collide. (5 marks)

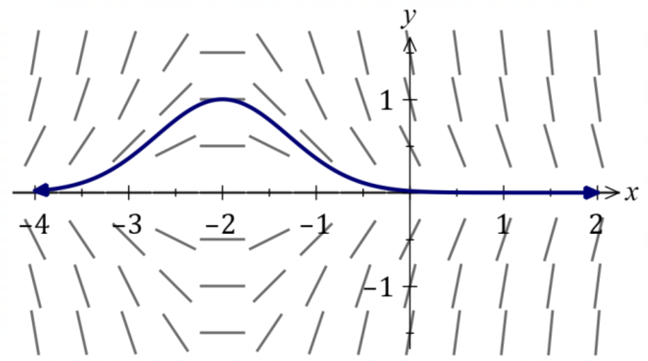
Solution
Velocity of Q is $\frac{1}{2}\left(\begin{pmatrix} 6 \\ 7 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ 11 \\ -5 \end{pmatrix}\right) = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$. Position vector of Q, s seconds after $t = 0$ is: $$\underset{\sim}{r}_Q(s) = \begin{pmatrix} 4 \\ 11 \\ -5 \end{pmatrix} + s\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$$ For collision $\begin{pmatrix} 4 \\ 11 \\ -5 \end{pmatrix} + s\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ -7 \\ 8 \end{pmatrix} + t\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$. Equating $\underset{\sim}{i}$ and $\underset{\sim}{j}$ coefficients: $$4 + s = -2 + 2t$$ $$11 - 2s = -7 + 2t$$ Solving these equations simultaneously we get $s = 4, t = 5$. $$\underset{\sim}{r}_Q(4) = \begin{pmatrix} 4 \\ 11 \\ -5 \end{pmatrix} + 4\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 8 \\ 3 \\ 3 \end{pmatrix}$$ $$\underset{\sim}{r}_P(5) = \begin{pmatrix} -2 \\ -7 \\ 8 \end{pmatrix} + 5\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 8 \\ 3 \\ 3 \end{pmatrix}$$ Since particles P and Q both pass through $(8, 3, 3)$ but at different times then their paths cross but they do not collide.
Specific behaviours
✓ obtains velocity vector for Q ✓ obtains position vector for Q ✓ equates two pairs of coefficients ✓ solves simultaneously ✓ shows that particles have same position vectors at different times

Question 14 **(7 marks)**

The slope field for the differential equation

$$\frac{dy}{dx} + y(2x + k) = 0$$

where k is a constant, is shown at right.



Solution (c)
See graph
Specific behaviours
✓ 'normal' curve thru' $(-2, 1)$

(a) Use a feature of the slope field to explain why $k = 4$ and hence determine the slope at the point $A(-3, -1)$. (2 marks)

Solution
$y' = -y(2x + k)$ When $x = -2$ and $y \neq 0$ it can be seen that $y' = 0$ and so $2(-2) + k = 0 \Rightarrow k = 4$. $A(-3, -1) \rightarrow y' = -(-1)(2(-3) + 4) = -2$. Slope at $A(-3, -1)$ is -2.
Specific behaviours
✓ explains using $y' = 0$ at $x = -2$ ✓ correct slope at A

(b) Determine the solution of the differential equation that contains the point $B(-2, 1)$ in the form $y = f(x)$. (4 marks)

Solution
$\frac{dy}{dx} = -y(2x + 4)$ $\int \frac{1}{y} dy = -\int 2x + 4\, dx$ $\ln\lvert y \rvert = -x^2 - 4x + c$ At $B(-2, 1)$, $y > 0$ and so require $\ln(y) = -x^2 - 4x + c$ $-(-2)^2 - 4(-2) + c = 0 \Rightarrow c = -4$ $y = e^{-x^2-4x-4} = e^{-(x+2)^2}$ $(\approx 0.0183e^{-x^2-4x})$
Specific behaviours
✓ separates variables and antidifferentiates ✓ recognises that $y > 0$ to replace $\lvert y \rvert \rightarrow (y)$ ✓ evaluates constant ✓ correctly expresses y as a function of x

(c) Sketch the solution curve that contains the point $B(-2, 1)$ on the slope field. (1 mark)

Question 15 **(6 marks)**

Consider the function $f(x) = \dfrac{ax^2 + 2ax - b}{x^2 - c}$, where a, b and c are positive constants.

The graph of $y = f(x)$ cuts the x-axis at $x = -4$, has a horizontal asymptote with equation $y = 3$ and has a vertical asymptote with equation $x = -3$.

(a) Determine $f(0)$. (3 marks)

Solution
Horizontal asymptote $y = 3 \Rightarrow a = 3$.
$f(-4) = 0 \Rightarrow 3(-4)^2 + 2(3)(-4) - b = 0 \Rightarrow b = 24$
Vertical asymptote $x = -3 \Rightarrow (-3)^2 - c = 0 \Rightarrow c = 9$
$f(0) = \dfrac{-24}{-9} = \dfrac{8}{3}$
Specific behaviours
✓ obtains value of one constant
✓ obtains value of second constant
✓ correct value of $f(0)$

(b) Now consider the graph of $y = \dfrac{1}{f(x)}$. State the

(i) equation of its horizontal asymptote. (1 mark)

Solution
$y = \dfrac{1}{3}$
Specific behaviours
✓ correct equation

(ii) x-axis intercepts. (1 mark)

Solution
Vertical asymptotes → roots: $x = \pm 3$.
Specific behaviours
✓ correct intercepts

(iii) equations of its vertical asymptotes. (1 mark)

Solution
Roots → vertical asymptotes: $3x^2 + 6x - 24 = 0 \Rightarrow x = -4$ and $x = 2$.
Specific behaviours
✓ correct equations

Question 16 **(8 marks)**

A machine fills bags with sugar. The mean and standard deviation of the weight of sugar it delivers into a bag is 505 and 17 grams respectively. An inspector routinely takes a random sample of 76 bags filled by the machine.

(a) For repeated random sampling of 76 bags of sugar filled by this machine, state the approximate distribution of the sample mean that the inspector should expect. (3 marks)

Solution
Let $\bar{X}$ be the sample mean. Since the sample size is large then the distribution of $\bar{X}$ will be approximately normal with mean 505 g.
The standard deviation of $\bar{X}$ is $\frac{17}{\sqrt{76}} = 1.950$ grams (variance≈ 3.8)
Hence $\bar{X} \sim N(505, 1.95^2)$.
Specific behaviours
✓ states that sample mean will be normally distributed ✓ states the mean of the distribution ✓ states the variance or standard deviation of the distribution

(b) Determine the probability that the mean weight of a random sample of 76 bags of sugar is at least 502 grams, given that the sample mean is less than 505 grams. (2 marks)

Solution
$P(\bar{X} > 502 \mid \bar{X} < 505) = \dfrac{P(502 < \bar{X} < 505)}{P(\bar{X} < 505)} = \dfrac{0.438}{0.5} = 0.876$
Specific behaviours
✓ forms correct probability statement ✓ correct probability

(c) Occasionally, the inspector only has enough time to take a random sample of 50 bags. In the long run, 80% of sample means derived from samples with this smaller size will lie in the range $505 \pm k$ grams. Determine the value of k. (3 marks)

Solution
The new standard deviation of $\bar{X}$ is $\frac{17}{\sqrt{50}} = 2.404$ grams (variance$= 5.78$).
$\bar{X} \sim N(505, 5.78)$
$P(505 - k < \bar{X} < 505 + k) = 0.8$
$k = 3.08\text{ g}$
Specific behaviours
✓ states new parameters of distribution of sample mean ✓ writes correct probability statement ✓ correct value of k

Question 17 **(7 marks)**

(a) Use the substitution $u^2 = 3y + 10$ to show that $\displaystyle\int \frac{y}{\sqrt{3y+10}}\,dy = \frac{(6y-40)\sqrt{3y+10}}{27} + c$,

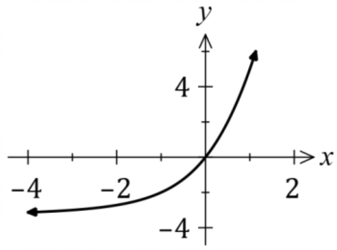
where c is a constant of integration. (4 marks)

Solution
$u^2 = 3y + 10 \Rightarrow dy = \frac{2u}{3}du, \qquad y = \frac{u^2-10}{3}$ $\int \frac{y}{\sqrt{3y+10}}\,dy = \int \frac{u^2-10}{3u} \times \frac{2u}{3}\,du$ $= \frac{2}{9}\int u^2 - 10\,du$ $= \frac{2}{9}\left(\frac{1}{3}u^3 - 10u\right) + c$ $= \frac{2u}{27}(u^2-30) + c$ $= \frac{2\sqrt{3y+10}(3y+10-30)}{27} + c$ $= \frac{(6y-40)\sqrt{3y+10}}{27} + c$
Specific behaviours
✓ obtains y and dy in terms of u and du ✓ obtains simplified integral in terms of u ✓ obtains correct antiderivative with constant   ✓ shows step(s) that clearly lead to required result

(b) The equation of the curve shown is

$y = x\sqrt{3y+10}$.

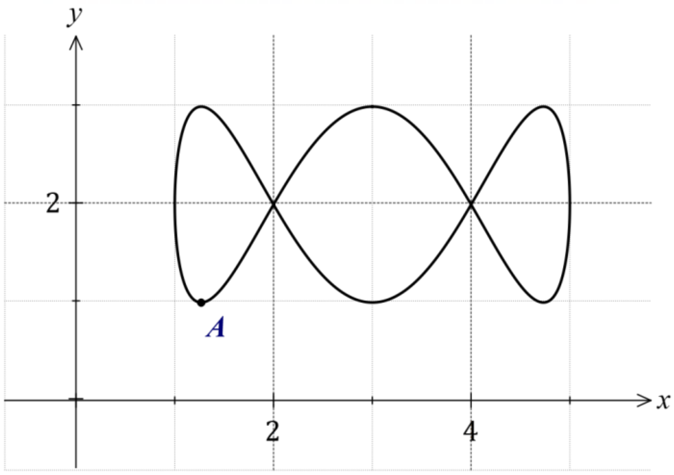
Determine the area enclosed by the curve and the line $2x - y + 3 = 0$. (3 marks)



Solution
$x = \frac{y}{\sqrt{3y+10}}, \qquad x = \frac{y-3}{2}$ Lines intersect when $y = -3, y = 5$. $A = \int_{-3}^{5} \left(\frac{y}{\sqrt{3y+10}}\right) - \left(\frac{y-3}{2}\right) dy$ $= \frac{224}{27} \approx 8.296$
Specific behaviours
✓ obtains bounds of integral   ✓ writes correct integral for area   ✓ correct area

Question 18 **(9 marks)**

A particle is moving and has position vector $\underset{\sim}{r}(t) = \begin{pmatrix} 3 - 2\sin(t) \\ 2 + \cos(3t) \end{pmatrix}$ metres, where t is the time in seconds since motion began. Its path is shown in the diagram below.



(a) Mark point A on the diagram above to show the position of the particle when $t = \frac{\pi}{3}$, and state the time taken for the particle to next return to this position. (2 marks)

Solution
Locate point at $A\left(3 - \sqrt{3}, 1\right) \approx (1.27, 1)$. Using period of $\sin t$, the particle will return here after 2π seconds.
Specific behaviours
✓ correctly marks point ✓ states period of motion

(b) Determine the velocity of the particle when $t = \frac{\pi}{3}$. (2 marks)

Solution
$\underset{\sim}{v}(t) = \frac{d}{dt}\underset{\sim}{r}(t)$ $= \begin{pmatrix} -2\cos(t) \\ -3\sin(3t) \end{pmatrix}$ $\underset{\sim}{v}\left(\frac{\pi}{3}\right) = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$
Specific behaviours
✓ obtains correct velocity vector ✓ correct velocity

(c) Determine the distance moved by the particle during its fourth second of motion. (2 marks)

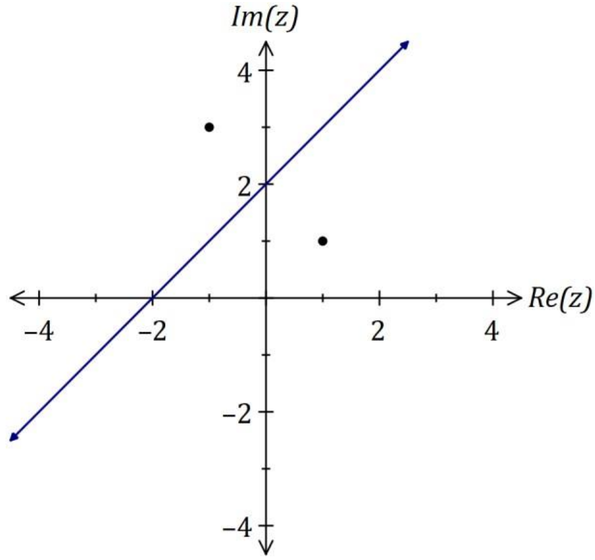
Solution
Let $s = \left\lvert \underset{\sim}{v}(t) \right\rvert = \sqrt{4\cos^2 t + 9\sin^2 3t}$ Then distance is: $$d = \int_3^4 s\,dt \approx 2.76\text{ m}$$
Specific behaviours
✓ indicates correct integral for distance ✓ correct distance

(d) Using the identity $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$, or otherwise, determine the Cartesian equation of the path of the particle. (3 marks)

Solution
$$x = 3 - 2\sin t \Rightarrow \sin t = \frac{3-x}{2}$$ $$\begin{aligned} y &= 2 + \cos 3t \\ y - 2 &= 4\cos^3 t - 3\cos t \\ &= \cos t\,(4\cos^2 t - 3) \\ &= \cos t\,(4(1-\sin^2 t) - 3) \\ &= \cos t\,(1 - 4\sin^2 t) \\ \cos t &= \frac{y-2}{1-4\left(\frac{3-x}{2}\right)^2} \\ &= \frac{y-2}{1-(3-x)^2} \end{aligned}$$ Hence $$\left(\frac{3-x}{2}\right)^2 + \left(\frac{y-2}{1-(3-x)^2}\right)^2 = 1$$
Specific behaviours
✓ obtains Cartesian expression for $\sin t$ ✓ obtains Cartesian expression for $\cos t$ ✓ combines to obtain correct Cartesian equation

Question 19 **(9 marks)**

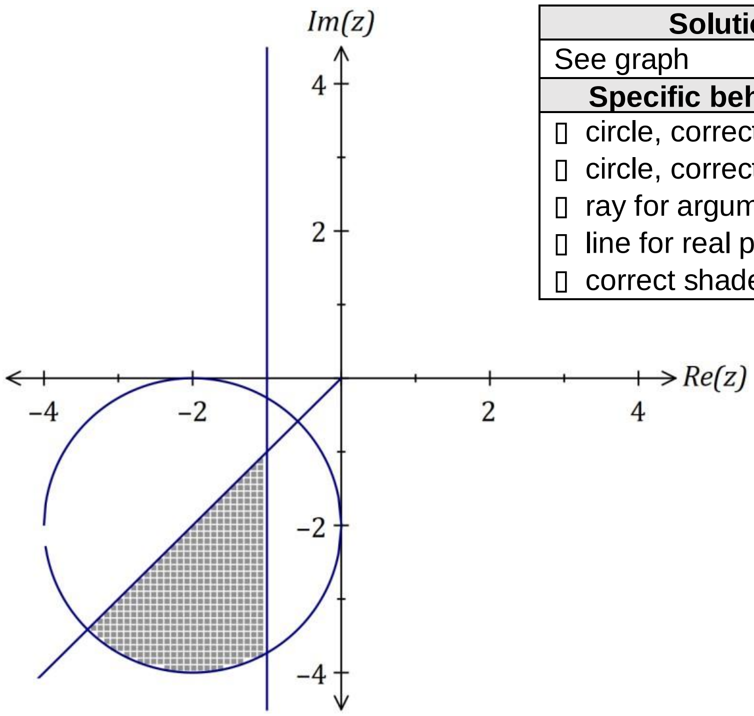
(a) On the Argand plane below, sketch the locus of $|z - 1 - i| = |z + 1 - 3i|$, where z is a complex number. (3 marks)



Solution	
See graph	
Specific behaviours	
✓	indicates $(1, 1)$ and $(-1, 3)$
✓	indicates perpendicular bisector
✓	correct axes intercepts

(a) Consider the three inequalities $|z+2+2i| \leq 2$, $\arg(z) \geq \frac{-3\pi}{4}$ and $\text{Re}(z) \leq -1$.

(i) On the Argand plane below, shade the region that represents the complex numbers satisfying these inequalities. (5 marks)



Solution
See graph
Specific behaviours
✓ circle, correct centre
✓ circle, correct radius
✓ ray for argument
✓ line for real part
✓ correct shaded region

(ii) Determine the minimum possible value of $\mathrm{Re}(z)$ within the shaded region. (1 mark)

Solution
$-2-2\sin\frac{\pi}{4}=-2-\sqrt{2}$
Specific behaviours
correct value

End of questions

Supplementary page

Question number: _________

Supplementary page

Question number: _________

© 2023 WA Exam Papers. Kennedy Baptist College has a non-exclusive licence to copy and communicate this document for non-commercial, educational use within the school. No other copying, communication or use is permitted without the express written permission of WA Exam Papers. SN245-224-4.